

1 Solve the equation $e^{2x} - 5e^x = 0$. [4]

2 The temperature T in degrees Celsius of water in a glass t minutes after boiling is modelled by the equation $T = 20 + be^{-kt}$, where b and k are constants. Initially the temperature is 100°C , and after 5 minutes the temperature is 60°C .

(i) Find b and k . [4]

(ii) Find at what time the temperature reaches 50°C . [2]

3 A piston can slide inside a tube which is closed at one end and encloses a quantity of gas (see Fig. 4).

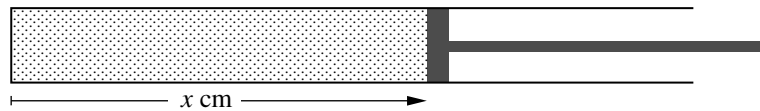


Fig. 4

The pressure of the gas in atmospheric units is given by $p = \frac{100}{x}$, where x cm is the distance of the piston from the closed end. At a certain moment, $x = 50$, and the piston is being pulled away from the closed end at 10 cm per minute. At what rate is the pressure changing at that time? [6]

4 A radioactive substance decays exponentially, so that its mass M grams can be modelled by the equation $M = Ae^{-kt}$, where t is the time in years, and A and k are positive constants.

(i) An initial mass of 100 grams of the substance decays to 50 grams in 1500 years. Find A and k . [5]

(ii) The substance becomes safe when 99% of its initial mass has decayed. Find how long it will take before the substance becomes safe. [3]

- 5 The profit £ P made by a company in its n th year is modelled by the exponential function

$$P = Ae^{bn}.$$

In the first year (when $n = 1$), the profit was £10 000. In the second year, the profit was £16 000.

- (i) Show that $e^b = 1.6$, and find b and A . [6]
- (ii) What does this model predict the profit to be in the 20th year? [2]

- 6 In a chemical reaction, the mass m grams of a chemical after t minutes is modelled by the equation

$$m = 20 + 30e^{-0.1t}.$$

- (i) Find the initial mass of the chemical.

What is the mass of chemical in the long term? [3]

- (ii) Find the time when the mass is 30 grams. [3]
- (iii) Sketch the graph of m against t . [2]

- 7 The value £ V of a car is modelled by the equation $V = Ae^{-kt}$, where t is the age of the car in years and A and k are constants. Its value when new is £10 000, and after 3 years its value is £6000.

- (i) Find the values of A and k . [5]
- (ii) Find the age of the car when its value is £2000. [2]